

Constraints on R -parity violating interactions from $\mu \rightarrow e\gamma$

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Abstract

The lepton number violating process $\mu \rightarrow e\gamma$ is used to study the lepton number violating couplings in MSSM extended by terms violating R -parity explicitly. Bounds are obtained for the products of λ - or λ' -type couplings. It is found that many of these limits are more stringent than the ones obtained previously.

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The decay of muon to electron and photon is a useful process to test theories with lepton number violating interactions, since it is experimentally very strictly constrained. In the context of supersymmetry, this process has been studied earlier in the Minimal Supersymmetric Standard Model (MSSM) assuming either universality of the soft scalar masses at the GUT scale [1] or letting the non-diagonal scalar masses or trilinear soft couplings to be arbitrary [2]. In the first case it was found that the non-universality coming from the RGE evolution from GUT to weak scale is too small to be observed, while in the second case the larger the non-diagonal terms at the GUT scale, the heavier the spectrum of the supersymmetric partners should be to satisfy the experimental bounds. Here we will assume that the contribution from mixing of the slepton generations is negligible and look for other possible sources of lepton number violation.

In the MSSM, the conservation of lepton and baryon number in the Standard Model has been put in by conservation of the so-called R -parity, $R = (-1)^{3(B-L)+2S}$. The supersymmetry and gauge invariance allow also R -violating interactions, namely

$$W_R = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \lambda''_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c - \mu_i L_i H_2. \quad (1)$$

Here $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. The λ , λ' and μ terms violate the lepton number, whereas the λ'' terms violate the baryon number by one unit. We assume that the μ terms can be rotated away by a redefinition of fields [3, 4]. It is worth noting that models exist, in which the violation of R -parity is a necessity, e.g. in the supersymmetric left-right model the R -parity is automatically conserved at the level of the Lagrangian, but in the genuine minimum of the potential for this model one or more of the sneutrinos get a VEV and hence violate the lepton number and R -parity [5]. Simultaneous presence of L violating ($\lambda_{ijk}, \lambda'_{ijk}$) and B violating (λ''_{ijk}) couplings would generally lead to too fast proton decay. It was recently argued [6] that proton decay for squark masses below 1 TeV always constrains the product $|\lambda' \cdot \lambda''| < 10^{-9}$. Huge difference between the strengths of lepton and baryon number violating couplings are also supported by examples of GUT models, in which quarks and leptons are treated differently [3, 7]. Furthermore, it is well known that in string unification the conservation of R -parity is not evident [8]. In this paper it is assumed that only the lepton number violating couplings are non-vanishing.

The strength of the couplings in Eq. (1) has been studied from several different sources, e.g. charged current universality, $e - \mu - \tau$ universality, forward-backward asymmetry, $\nu_\mu e$ scattering, atomic parity violation [9], neutrinoless double beta decay [10], ν masses [11], heavy nuclei decay [13], Z -boson partial width [14], and K^+ , t -quark decays [15]. We have updated a table of limits for λ - and λ' -couplings, Table 1. The bounds are typically between $10^{-3} - 10^{-1}$ as seen from Table 1. Limits on

$\lambda_{ijk} <$	$m_{\tilde{f}} = 100\text{GeV}$	$\lambda'_{ijk} <$	$m_{\tilde{f}} = 100\text{GeV}$	$\lambda'_{ijk} <$	$m_{\tilde{f}} = 100\text{GeV}$
12k	0.04 [9]	111	0.0004 [10]	22k	0.012 [15]
131	0.10 [9]	112	0.03 [9]	231	0.22 [9]
132	0.10 [9]	113	0.03 [9]	232	0.44 [14]
133	0.001 [11]	12k	0.012 [15]	233	0.44 [14]
23k	0.09 [9]	131	0.26 [9]	31k	0.012 [15]
		132	0.51 [14]	32k	0.012 [15]
		133	0.001 [11]	33k	0.26 [14]
		21k	0.012 [15]		

Table 1: Previously found limits on single λ_{ijk} and λ'_{ijk} .

neutrino masses give stringent bounds for λ_{133} and λ'_{133} couplings due to tau lepton or bottom quark in the loop inducing Majorana mass term [11].

Severe limitations come also from baryogenesis considerations if it is required that the primordial baryon asymmetry is preserved, since L violating interactions in equilibrium together with $B+L$ violating sphalerons would wash out any pre-existing baryon asymmetry. The sphalerons preserve $\frac{1}{3}B - L_i$ and it has been shown in [16] that any bound from cosmology can be avoided by demanding that one of the lepton numbers is conserved. Since we wish to study the decay $\mu \rightarrow e\gamma$, we could have conservation of τ lepton number. On the other hand, if electroweak baryogenesis is assumed (see, e.g. [17]), the restrictions on lepton number are removed. We shall give our results both assuming τ number conservation and relaxing this assumption.

Addition of Eq. (1) to MSSM superpotential leads to more interactions in the model, but the MSSM particle content remains. The relevant part of the Lagrangian is found by the standard techniques [18]

$$\begin{aligned}
L_{\psi, \lambda, \lambda'} = & \lambda_{ijk}(\tilde{\nu}_{iL}\bar{e}_{kR}e_{jL} + \tilde{e}_{jL}\bar{e}_{kL}\nu_{iL} + \tilde{e}_{kR}^*\overline{\nu_{iL}^c}e_{jL} - (i \leftrightarrow j)) \\
& + \lambda'_{ijk}(-\tilde{u}_{jL}\bar{d}_{kR}e_{iL} - \tilde{d}_{kR}^*\overline{e_{iL}^c}u_{jL}) + h.c.,
\end{aligned} \tag{2}$$

from which the contributions to the radiative muon decay are found. These are shown in Fig. (1). The photon line is not shown, but it should be attached in all possible ways to the graphs. If τ number conservation is assumed, only those graphs which have in the loop none or two particles carrying τ number should be included.

The gauge invariant amplitude for $\mu \rightarrow e\gamma$ is usually parametrized as

$$T(\mu \rightarrow e\gamma) = \epsilon^\lambda \bar{u}_e(p')(A + B\gamma_5)i\sigma_{\lambda\nu}q^\nu u_\mu(p), \tag{3}$$

where p , p' and q are the momenta of muon, electron and photon, respectively. ϵ^λ is the photon polarization vector and $\sigma_{\lambda\nu} = \frac{i}{2}[\gamma_\lambda, \gamma_\nu]$. It is easily seen from Eq. (3), that

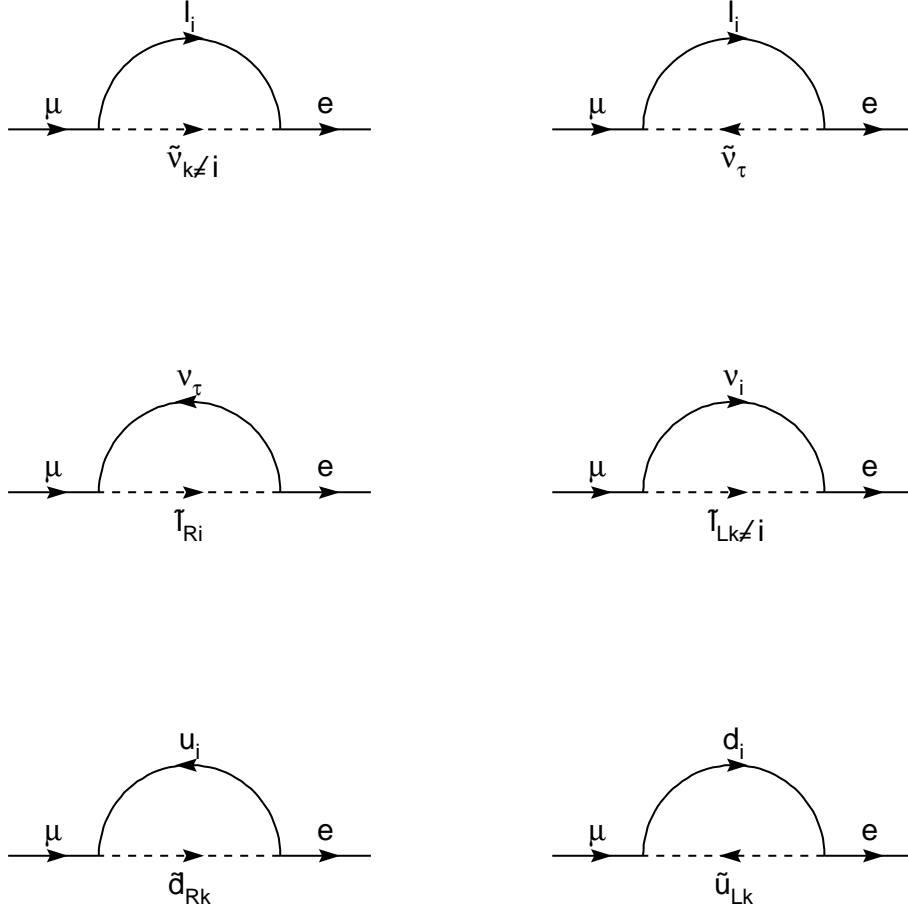


Figure 1: Contributing diagrams for the muon decay to electron and photon. Unless otherwise indicated $i, k = 1 \dots 3$.

the amplitude is nonvanishing only when the muon and the electron are of opposite helicities.

The width of the decay can be evaluated using the amplitude, Eq. (3), and finally the branching ratio from μ lifetime, $\tau_\mu = 192\pi^2/(G_F^2 m_\mu^5)$, with the result

$$BR = \frac{24\pi}{G_F^2 m_\mu^2} (|A|^2 + |B|^2). \quad (4)$$

Similarly one could study the decay of tau lepton to muon and photon or electron and photon.

The experimental limits for the lepton decays are given as [19]

$$\begin{aligned}
BR(\mu \rightarrow e\gamma) &< 4.9 \cdot 10^{-11}, \\
BR(\tau \rightarrow \mu\gamma) &< 4.2 \cdot 10^{-6}, \\
BR(\tau \rightarrow e\gamma) &< 1.2 \cdot 10^{-4}.
\end{aligned} \tag{5}$$

Using Gordon decomposition for practical calculations, the relevant part of all the amplitudes corresponding to the graphs in Fig. (1) are found. A in the amplitude, Eq. (3), can be written in terms of the following functions:

$$\begin{aligned}
A_1 &= \frac{\lambda_1 \lambda_2 Q m_\mu}{16\pi^2 m_{\tilde{f}}^2} \frac{1}{6(\kappa - 1)^3} \left(-\kappa^2 + 5\kappa + 2 - \frac{6\kappa}{\kappa - 1} \ln \kappa \right), \\
A_2 &= \frac{\lambda_1 \lambda_2 Q m_\mu}{16\pi^2 m_{\tilde{f}}^2} \frac{1}{6(\kappa - 1)^3} \left(2\kappa^2 + 5\kappa - 1 - \frac{6\kappa^2}{\kappa - 1} \ln \kappa \right).
\end{aligned} \tag{6}$$

There are two lepton number violating vertices in every contribution, characterized by $\lambda_{1,2} = \lambda_{i,j,k}$ or $\lambda_{1,2} = \lambda'_{i,j,k}$ couplings. The functions A_1 and A_2 depend also on the charge of the particle attached to the photon Q , the sfermion mass occurring in the loop $m_{\tilde{f}}$, and the ratio of the masses of fermion and sfermion in the loop, $\kappa = m_f^2/\tilde{m}_f^2$. Proportionality to the mass of the muon, m_μ , reflects the helicity flip on the external muon line. The lack of a term proportional to the electron mass on the other hand indicates that we have approximated the external electron to be massless. The A_1 function corresponds to the situation where the photon line is attached to the fermion line and A_2 to the situation where the photon line is attached to the scalar. In all cases $|A| = |B|$ in Eq. (3).

We have analyzed three cases, namely i) the λ couplings dominate and are the same, $\lambda_{ijk} = \lambda$, ii) the λ' couplings dominate and are the same, $\lambda'_{ijk} = \lambda'$, iii) one pair of the λ or λ' couplings dominates over the others.

In cases i) and ii), the spectrum is calculated by assuming universal scalar mass and universal gaugino mass at the GUT scale 10^{16} GeV and evaluating the masses down to the electroweak scale [21]. Two sets of universal mass parameters are used, one leading to a light SUSY spectrum (slepton and squark masses between 100–400 GeV) and another one leading to heavy SUSY spectrum with masses 1–1.4 TeV. In Figs. (2) a) and c), the solid lines give the upper limits for the couplings when τ number is conserved. If τ number conservation is relaxed (dashed lines), more graphs contribute and limit for the couplings become stricter. From Figs. (2) a) and c) it is seen that when the spectrum is light, the maximum coupling is $\mathcal{O}(10^{-2})$. In the case of the heavy spectrum, Figs. (2) b) and d), the maximum coupling is approximately one third of the electromagnetic coupling.

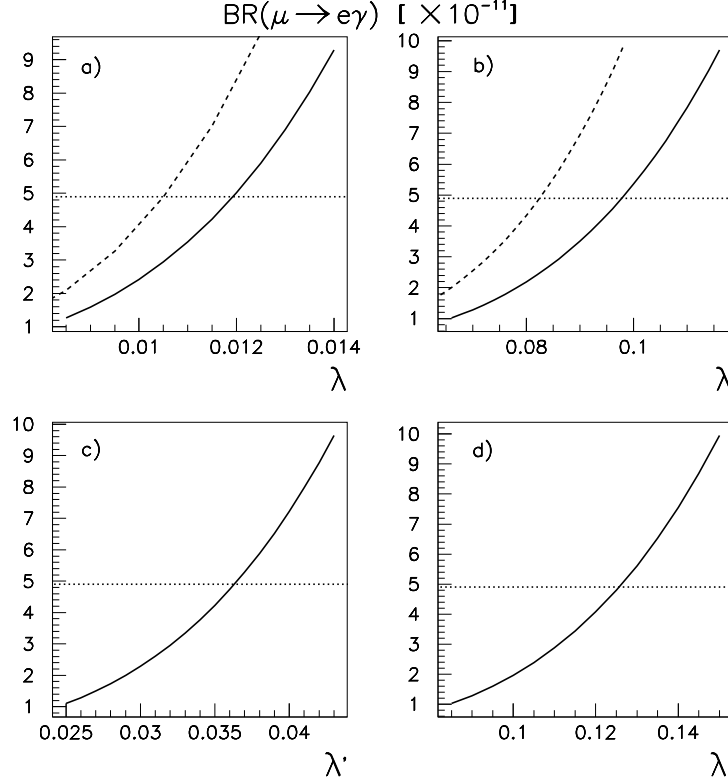


Figure 2: The branching ratio for $\mu \rightarrow e\gamma$, when a) and b): $\lambda_{ijk} = \lambda$ and $\lambda'_{ijk} = 0$ for all i, j, k and c) and d): $\lambda'_{ijk} = \lambda'$ and $\lambda_{ijk} = 0$ for all i, j, k . In a) and c) the SUSY spectrum is light and in b) and d) the SUSY spectrum is heavy. The dashed lines in a) and b) correspond to the situation in which τ number conservation has been relaxed. The dotted line is the experimental limit, $BR(\mu \rightarrow e\gamma) < 4.9 \cdot 10^{-11}$.

In case iii), a large hierarchy between various pairs of λ or λ' couplings has been assumed. Case iii) is in a sense a natural choice, since the λ and λ' couplings are similar to the Yukawa couplings, which are known to vary over at least six orders of magnitude. This is also the most conservative limit for the couplings. This case has been studied for two different scalar masses in the loop, namely $m_{\tilde{f}} = 100$ GeV and $m_{\tilde{f}} = 1$ TeV. For the lighter scalar mass, one sees from Table 2 that the last three limits are much less strict than the others. This is due to the top quark ($m_{top} \sim 175$ GeV) in the loop, since then the value of κ is larger than one. Also the effect of the bottom quark is seen in some of the bounds, especially in the last one, when both top- and bottom-quarks contribute to the result. When the scalar mass in the loop is 1 TeV, one does not anymore see the effect of the bottom quark and also the effect

	$m_{\tilde{f}} = 100 \text{ GeV}$ $\times 10^{-4}$	$m_{\tilde{f}} = 100 \text{ GeV}$ previous results $\times 10^{-4}$	$m_{\tilde{f}} = 1 \text{ TeV}$ $\times 10^{-2}$
$ \lambda_{121} \lambda_{122} <$	1.0	16	1.0
$ \lambda_{131} \lambda_{132} <$	1.0	100	1.0
$ \lambda_{231} \lambda_{232} <$	1.0	81	1.0
$ \lambda_{231} \lambda_{131} <$	2.0	90	2.0
$ \lambda_{232} \lambda_{132} <$	2.0	90	2.0
$ \lambda_{233} \lambda_{133} <$	2.0	0.9	2.0
$ \lambda'_{211} \lambda'_{111} <$	8.0	.048	8.0
$ \lambda'_{212} \lambda'_{112} <$	8.0	1.4	8.0
$ \lambda'_{213} \lambda'_{113} <$	8.1	1.4	8.0
$ \lambda'_{221} \lambda'_{121} <$	8.1	1.4	8.0
$ \lambda'_{222} \lambda'_{122} <$	8.1	1.4	8.0
$ \lambda'_{223} \lambda'_{123} <$	8.2	1.4	8.0
$ \lambda'_{231} \lambda'_{131} <$	140	570	10.4
$ \lambda'_{232} \lambda'_{132} <$	140	2200	10.4
$ \lambda'_{233} \lambda'_{133} <$	180	4.4	10.4

Table 2: Upper limits on products of λ_{ijk} and λ'_{ijk} couplings for two different scalar masses, in the first column $m_{\tilde{f}}=100 \text{ GeV}$ and in the third $m_{\tilde{f}}=1 \text{ TeV}$. The second column contains earlier results from Table 1.

due to the top quark is much less prominent.

Comparing these products with the earlier limits, Table 1, it is seen that only if the previous limit comes from the constraint on the neutrino mass, double β decay or K^+ meson decays, it is more stringent than the present one.

If the τ number were not broken, the limits in Table 2 on λ type couplings should be included only when none or two of the i, j, k are 3's. One could also study the limits from $\tau \rightarrow \mu\gamma$ or $\tau \rightarrow e\gamma$, but as it appears these limits would not be tight enough to strengthen the bounds found previously.

To summarize, we have shown that the experimental upper limit on the muon radiative decay can be used to obtain stringent bounds on the magnitude of the R -parity violating interactions. Although we have considered the case of explicit R -parity breaking, the case of spontaneous breaking of this symmetry can be treated by a similar analysis. The interplay between the dominance of cross sections for reactions with exact or broken R -parity implemented by such bounds, is of great interest in the search for supersymmetric particles.

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